Softening of the phase transition in a two-dimensional Potts model under quenched bond randomness

Fatih Yaşar, Yiğit Gündüç, and Tarık Çelik
Physics Department, Hacettepe University, Beytepe 06532, Ankara, Turkey
(Received 3 April 1998)

We have simulated, by using a cluster algorithm, the $q=8$ state Potts model in two dimensions with a varying amount of quenched bond randomness. We have shown that there exists a finite size-dependent threshold value of the introduced quenched bond randomness for rounding the first-order phase transition and that this threshold value becomes smaller as the system size increases. [S1063-651X(98)15509-5]

PACS number(s): 64.60.Cn, 05.70.Jk, 75.10.Nr

The effect of the addition of quenched bond randomness to classical systems whose pure version displays a continuous phase transition is well understood in terms of the Harris conjecture [1], which states that the values of the critical exponents change only if the specific heat exponent α is positive. Recently, the introduction of randomness to the systems undergoing a first-order phase transition has attracted much interest. Phenomenological renormalization group arguments by Hui and Berker [2] and rigorous proof of the vanishing of the latent heat by Aizenman and Wehr [3] showed that bond randomness will induce a second-order phase transition in a system that would otherwise undergo a symmetry-breaking first-order phase transition. The renormalization group (nonrigorous) arguments state that any infinitesimal amount of bond randomness will drive the system to possess a second-order phase transition for $d \leqslant 2$ where d is the dimension of space. The numerical evidence for this prediction is provided by Monte Carlo simulation of the two-dimensional eight-state Potts model [4] under a certain type of bond randomness [5]. While the pure model is known to possess a strong first-order phase transition, the simulations with two sets of randomly distributed ferromagnetic bonds of two different strengths of certain ratio gave clear evidence of a second-order transition with 2D Ising exponents. For the convenience of establishing the histograms, the strong to weak bond ratio was chosen as $r=2$ or 10 in Ref. [5] and the simulations were carried out on lattices of linear size $12 \leqslant L \leqslant 128$. In this context, it would be quite relevant to study the changes in the characteristic behaviors of the system with respect to the introduction of a gradually increasing degree of bond randomness as well as its finite size dependence.

Distinguishing the order of a phase transition is one of the major problems concerning computer simulation studies of spin systems. The major difficulty, in this respect, arises when the correlation length is finite but larger than the lattice size. In such situations, commonly used tools of identifying the order of a transition (e.g., by looking for the minima in free energy [6] or by considering the probability distributions of energy [7]) may not be indicative. Even if there exist metastable states, the size of the system may prevent the observation of the double-peak behavior in the energy distribution. The basic reason for such behavior may be that energy is a local quantity. Quantities of global nature are expected to be more sensitive to the correlation length and hence the effects of having metastable states may be more emphasized. In an earlier work [8,9] on a two-dimensional Potts model, it was observed that, compared to the local operators such as energy and order parameter, cluster related (global) operators are more sensitive to structural changes in the system going through a phase transition. Particularly, the average cluster size distribution may give a better indication of the order of the phase transition at smaller lattice sizes than that for the energy distribution. The average cluster size can be defined as

$$ACS = \frac{1}{N_C} \left\langle \sum_{i=1}^{N_C} C_i \right\rangle, \tag{1}$$

where N_C is the number of clusters and C_i is the number of spins in the ith cluster normalized by dividing by the total number of spins.

The aim in this work is to simulate, by using a cluster algorithm, the two-dimensional q-state Potts model with a varying quenched bond randomness and to observe the energy and the average cluster size histograms in order to gain insight into the order of the phase transition in the system.

The Hamiltonian of the Potts model [10] is given by

$$\mathcal{H} = \sum_{\langle i,j \rangle} K_{ij} \delta_{\sigma_i, \sigma_j}. \tag{2}$$

Here $K_{ij} = J_{ij}/kT$, where k and T are the Boltzmann constant and the temperature, respectively, and J_{ij} is the mag-

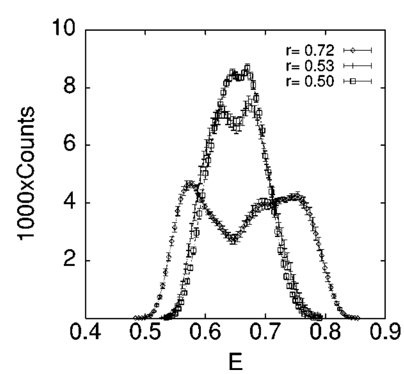

FIG. 1. Energy histograms for several values of r on a 64×64 lattice.

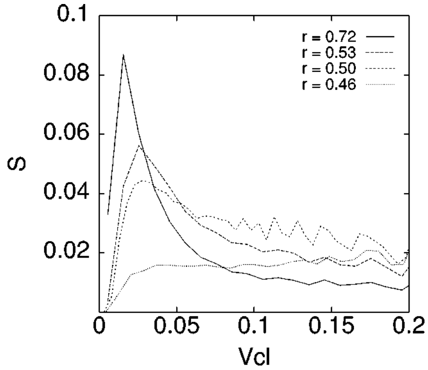

FIG. 2. Averaged cluster size distributions for several values of r on a 64×64 lattice.

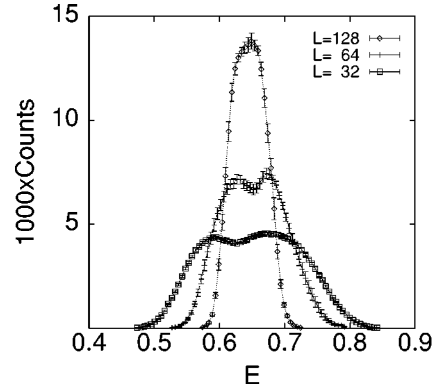

FIG. 3. Energy histograms for $r=0.53$ on several lattices.

netic interaction between spins σ_i and σ_j, which can take values $1,2,\ldots,q$ for the q-state Potts model. In the pure system, K_{ij} is a constant and the value of the transition temperature is known for all q as $K_c=\ln(1+\sqrt{q})$. In the random bond Potts model, the couplings are selected from two positive values with a weak to strong ratio $r=K_w/K_s$, which are chosen randomly from the distribution

$$P(K)=p\,\delta(K-K_w)+(1-p)\,\delta(K-K_s).$$

When the probability p is taken as $p=0.5$, the duality relation [11]

$$(e^{K_w^c}-1)(e^{K_s^c}-1)=q$$

gives the critical values K_w^c and K_s^c of the corresponding couplings K_w and K_s.

The two-dimensional q-state Potts model is known to undergo a first-order phase transition for $q\geqslant5$ [12]. In our simulation of the eight-state Potts model, the measurements are done on three lattices of the sizes 32^2, 64^2, and 128^2. Since the calculated correlation length ξ of this model is about 20 lattice sites [13], these lattices are considered to be an indicative set for both large lattice measurements and for the lattices of the order of the correlation length. Measurements are done about the transition temperature and at each temperature 5×10^5 iterations are performed following the thermalization runs of 5×10^4 to 10^5 iterations. For all simulation works, the cluster update algorithm [14,15] is employed.

We have examined the phase structure of the eight-state Potts model with respect to the quenched randomness in the range starting from $r=1$ (the pure case) to $r=0.1$ (weak bonds are ten times weaker). For a fixed value of r, 20 replicas are created by randomly distributing the strong and weak bonds over the lattice. The histograms for energy and the average cluster size are obtained by averaging the corresponding quantities over the replicas. In Fig. 1, we show the energy histogram on 64×64 lattice for several values of r. Starting from the pure case, which exhibits a rather strong first-order phase transition, the eight-state Potts model with quenched bond randomness displays a double-peak structure in the energy histogram down to $r=0.5$; the latter value corresponds to the case studied in Ref. [5]. By further tuning, one observes a single Gaussian in the energy histogram for $r=0.46$, which Gaussian gets sharper as the value of r chosen smaller. In order to obtain better insight into the order of phase transition, we examined the average cluster size distributions, which were found to yield quite useful information concerning the order of the phase transition in the pure q-state Potts model phase [8,9]. As shown in Fig. 2, the apparent peaks in the small cluster region, which is an indication of having metastable state disappear for $r=0.46$. The phase transition studied here on 64×64 lattice changes from first-order to second-order somewhere in the range $0.46\leqslant r\leqslant0.50$, the exact value of which is not attempted here to be evaluated. The observed threshold value for the randomness is specific to the lattice size used. Therefore the next step is to investigate how this threshold value changes with the size of the lattice. Figure 3 displays the energy histograms for a fixed value of r, namely, $r=0.53$, obtained on different size lattices. Here, this value of r chosen because the system apparently undergoes a weak first-order phase transition for this amount of quenched randomness. According to the finite size scaling arguments conjectured on lattice, for a first-order phase transition, the valley between the double peaks in the energy histogram should become deeper and scale as the volume with the increasing lattice size. On the other hand, the opposite behavior appears in a second-order transition and the double peak structure in the energy distribution slowly changes to a single Gaussian as the lattice volume becomes larger. From the distributions shown in Fig. 3, one will observe a first-order phase transition on a 64×64 lattice while a second-order on 128×128 lattice. One will then need to introduce less amounts of randomness (r smaller than 0.53) to regain the double peak structure in the energy distribution on the 128×128 lattice. Apparently, the observed threshold value for the amount of quenched impurities for the characteristic behavior of the system to change from having a first-order phase transition to a second-order one is specific to the finite size of the system under consideration. This threshold gets smaller and smaller as one moves towards the infinite systems, which is what we expect according to the renormalization group arguments put forward by Hui and Berker [2].

We thank W. Janke and D. Stauffer for fruitful discussions and their valuable comments. The hospitality at ZIF, University of Bielefeld, where part of this work was done, is gratefully acknowledged. This project is partially supported by Hacettepe University Research Fund under Project No. 95.010.10.003.

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